The Gruenberg-Kegel graph of finite solvable cut groups

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a joint work with

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Abstract

The Gruenberg-Kegel graph (abbreviated GK-graph) of a group G is the indirected graph with vertices the prime integers which occurs as the order of an element of G, and two different vertices p and q are join by an edge if G has an element of order $p \cdot q$. The GK-graph of G provides relevant information of G. Motivated by the First Zassenhaus Conjecture, Kimmerle proposed a weaker problem asking wether a finite group G has the same GK-graph than the group of normalized units of its integral group ring and proved that the answer is positive for solvable groups [2].

A fundamental group theoretical problem consists in determining which graphs occurs as GK-graphs of groups in a certain family. For example every graph is the GK-graph of some group but the GK-graphs of finite groups have at most 6 connected components [3, 4]. The GK-graphs of finite solvable groups has been recently classified in [1].

A finite group G is said to be cut if every **c**entral **u**nit of its integral group ring is **t**rivial i.e. belong to $\pm G$, or equivalently if for every irreducible character χ of G there is an imaginary quadratic extension of the rationals containing $\chi(G)$. In particular every rational group is cut. However while the class of rational groups is rather small, the cut groups are abundant. For example, of the groups of order at most 512 only 0,57% are rational while 86,62% are cut. In this talk we will present some recent result on the GK-graphs of solvable cut groups.

Key words

Gruenberg-Kegel graph.

References

[1] A. Gruber. T. M. Keller, M. L. Lewis, K. Naughton, and B. Strasser: *A characterization of the prime graphs of solvable groups.* J. Algebra 442 (2015), 397–422.

[2] W. Kimmerle, On the prime graph of the unit group of integral group rings if finite groups, Groups, rings and algebras, Contemp. Math., vol. 420, Amer. Math. Soc. Providence, RI, 2006, pp. 215–228.

[3] A. S. Kondrat'ev, On prime graph components of finite simple groups, Math. USSR Sbornik 67 (1990), no. 1, 235–247.

[4] J. S. Williams, *Prime graphs components of finite groups*, J. Algebra, 69 (1981), 487–513.